

SERIE 6 CORRECTION

3APIC

EXERCICE 1:

1) *Simplifier et calculer* : $A = \sqrt{49}$; $B = \sqrt{100} - \sqrt{121}$; $C = \sqrt{144} + \sqrt{36}$

2) *Ecrire sous la forme $a\sqrt{b}$ tel que b soit le plus petit possible* :

$$D = 2\sqrt{20} \quad ; \quad E = \sqrt{72} - \sqrt{32} \quad ; \quad F = \sqrt{3^3 \times 2^2} \quad ; \quad G = 2\sqrt{50} - \sqrt{18} + \sqrt{8}$$

$$E = \sqrt{72} - \sqrt{32} - 7\sqrt{2}$$

CORRECTION :

1) Simplifions et calculons :

$$A = \sqrt{49} = \sqrt{7^2} = 7 \quad ; \quad B = \sqrt{100} - \sqrt{121} = 10 - 11 = -1 \quad ; \quad C = \sqrt{144} + \sqrt{36} = 12 + 6 = 18$$

2) Ecrire sous la forme $a\sqrt{b}$ tel que b soit le plus petit possible :

$$D = 2\sqrt{20} = 2\sqrt{4 \times 5} = 2\sqrt{4} \times \sqrt{5} = 4\sqrt{5} \quad ; \quad F = \sqrt{3^3 \times 2^2} = \sqrt{3^2 \times 3} \times \sqrt{2^2} = 3\sqrt{3} \times 2 = 6\sqrt{3} \quad ;$$

$$E = \sqrt{72} - \sqrt{32} - 7\sqrt{2} = \sqrt{36 \times 2} - \sqrt{16 \times 2} - 7\sqrt{2} = 6\sqrt{2} - 4\sqrt{2} - 7\sqrt{2} = 2\sqrt{2} - 7\sqrt{2} = -5\sqrt{2}$$

$$G = 2\sqrt{50} - \sqrt{18} + \sqrt{8} = 2\sqrt{25 \times 2} - \sqrt{9 \times 2} + \sqrt{4 \times 2} = 2 \times 5\sqrt{2} - 3\sqrt{2} + 2\sqrt{2} = 10\sqrt{2} - 3\sqrt{2} + 2\sqrt{2} = 9\sqrt{2}$$

EXERCICE 2:

1) *Simplifier* :

$$A = \sqrt{2} \times \sqrt{8} \quad ; \quad B = \sqrt{4\sqrt{64} + 4} \quad ; \quad C = \sqrt{5+\sqrt{7}} \times \sqrt{5-\sqrt{7}}$$

2) *Ecrire sous la forme $a\sqrt{b}$ tel que b soit le plus petit possible* :

$$D = 2\sqrt{18} - \sqrt{8} - 4\sqrt{2} \quad ; \quad E = 3\sqrt{80} + \sqrt{125} - 2\sqrt{320}$$

CORRECTION :

1) Simplifions :

$$A = \sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8} = \sqrt{16} = 4 \quad ; \quad B = \sqrt{4\sqrt{64} + 4} = \sqrt{4 \times 8 + 4} = \sqrt{32 + 4} = \sqrt{36} = 6$$

$$C = \sqrt{5+\sqrt{7}} \times \sqrt{5-\sqrt{7}} = \sqrt{(5+\sqrt{7})(5-\sqrt{7})} = \sqrt{25-7} = \sqrt{18} = 3\sqrt{2}$$

2) Ecrivons sous la forme tel que soit le plus petit possible :

$$D = 2\sqrt{18} - \sqrt{8} - 4\sqrt{2} = 6\sqrt{2} - 2\sqrt{2} - 4\sqrt{2} = 0$$

$$E = 3\sqrt{80} + \sqrt{125} - 2\sqrt{320} = 3\sqrt{16 \times 5} + \sqrt{25 \times 5} - 2\sqrt{64 \times 5} = (12 + 5 - 16)\sqrt{5} = \sqrt{5}$$

EXERCICE 3:

1) *Rendre rationnel le dénominateur* :

$$F = \frac{2}{\sqrt{5}} - \frac{\sqrt{5}}{5} \quad ; \quad G = \frac{4\sqrt{5}}{\sqrt{3}} - \sqrt{\frac{15}{3}} \quad ; \quad H = \frac{3}{\sqrt{7}-1}$$

2) *Calculer* : $S = (\sqrt{2} + 2)^2 - \frac{2(4 + \sqrt{2})}{\sqrt{2}} \quad ; \quad T = (1 + \sqrt{3})^{-1} - (1 - \sqrt{3})^{-1}$

CORRECTION :

1) Rendre rationnel le dénominateur :

$$F = \frac{2}{\sqrt{5}} - \frac{\sqrt{5}}{5} = \frac{2 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} - \frac{\sqrt{5}}{5} = \frac{2\sqrt{5}}{5} - \frac{\sqrt{5}}{5} = \frac{2\sqrt{5} - \sqrt{5}}{5} = \frac{\sqrt{5}}{5}$$

$$G = \frac{4\sqrt{5}}{\sqrt{3}} - \sqrt{\frac{15}{9}} = \frac{4\sqrt{5} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} - \frac{\sqrt{15}}{\sqrt{9}} = \frac{4\sqrt{15}}{3} - \frac{\sqrt{15}}{3} = \frac{4\sqrt{15} - \sqrt{15}}{3} = \frac{3\sqrt{15}}{3} = \sqrt{15}$$

$$H = \frac{3}{\sqrt{7}-1} = \frac{3(\sqrt{7}+1)}{(\sqrt{7}-1)(\sqrt{7}+1)} = \frac{3(\sqrt{7}+1)}{(\sqrt{7})^2 - 1^2} = \frac{3(\sqrt{7}+1)}{7-1} = \frac{3(\sqrt{7}+1)}{6} = \frac{\sqrt{7}+1}{2}$$

2) Calculons:

$$S = \frac{1}{(\sqrt{2}+2)^{-2}} - \frac{2(4+\sqrt{2})}{\sqrt{2}} = (\sqrt{2}+2)^2 - \frac{2(4+\sqrt{2})\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = (\sqrt{2}+2)^2 - \frac{2(4+\sqrt{2})\sqrt{2}}{2}$$

$$= (\sqrt{2}+2)^2 - (4+\sqrt{2})\sqrt{2} = 2+4\sqrt{2}+4-4\sqrt{2}-2 = 4$$

$$T = (1+\sqrt{3})^{-1} - (1-\sqrt{3})^{-1} = \frac{1}{1+\sqrt{3}} - \frac{1}{1-\sqrt{3}} = \frac{(1-\sqrt{3}) - (1+\sqrt{3})}{1-3} = \frac{1-\sqrt{3}-1-\sqrt{3}}{-2} = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$$

EXERCICE 4:1) *Simplifier :*

$$H = \frac{1,5-6\sqrt{2}}{3} - \frac{3-4\sqrt{2}}{2} \quad ; \quad I = \frac{2\sqrt{3}-5}{2} + \frac{3\sqrt{3}+7}{5} - \frac{16\sqrt{3}-1}{10} \quad ; \quad J = \frac{5\sqrt{5}+2}{7} - \frac{2\sqrt{5}+3}{2} + \frac{4\sqrt{5}+3}{14}$$

2) *On pose : $B = \frac{5}{\sqrt{6}+1} - \frac{2\sqrt{3}}{\sqrt{2}}$. Montrer que B est un entier relatif.***CORRECTION:**

1) On trouve:

$$H = \frac{1,5-6\sqrt{2}}{3} - \frac{3-4\sqrt{2}}{2} = \frac{2(1-6\sqrt{2})-3(3-4\sqrt{2})}{3 \times 2} = \frac{3-12\sqrt{2}-9+12\sqrt{2}}{6} = \frac{-6}{6} = -1$$

$$I = \frac{2\sqrt{3}-5}{2} + \frac{3\sqrt{3}+7}{5} - \frac{16\sqrt{3}-1}{10} = \frac{5(2\sqrt{3}-5)+2(3\sqrt{3}+7)-(16\sqrt{3}-1)}{10} = \frac{10\sqrt{3}-25+6\sqrt{3}+14-16\sqrt{3}+1}{10} = \frac{-10}{10} = -1$$

$$J = \frac{5\sqrt{5}+2}{7} - \frac{2\sqrt{5}+3}{2} + \frac{4\sqrt{5}+3}{14} = \frac{2(5\sqrt{5}+2)-7(2\sqrt{5}+3)+4\sqrt{5}+3}{14} = \frac{10\sqrt{5}+4-14\sqrt{5}-21+4\sqrt{5}+3}{14} = \frac{-14}{14} = -1$$

2) On trouve:

$$B = \frac{5}{\sqrt{6}+1} - \frac{2\sqrt{3}}{\sqrt{2}} = \frac{5(\sqrt{6}-1)}{(\sqrt{6}+1)(\sqrt{6}-1)} - \frac{2\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{5(\sqrt{6}-1)}{6-1} - \frac{2\sqrt{6}}{2} = \sqrt{6}-1-\sqrt{6} = -1. \text{ Donc } B \text{ est un entier relatif.}$$

EXERCICE 5:*On pose: $I = (\sqrt{2}+3)^2$ et $J = (\sqrt{2}-2)(\sqrt{2}+3)$* 1) *Développer et réduire les expressions I et J .*2) *Montrer que: $I - J = 5\sqrt{2} + 15$.*3) *Montrer que: $I + J = (\sqrt{2}+3)(2\sqrt{2}+1)$.*

CORRECTION:

1) On a:

$$I = (\sqrt{2} + 3)^2 = (\sqrt{2})^2 + 2 \times 3\sqrt{2} + 3^2 = 2 + 6\sqrt{2} + 9 = 11 + 6\sqrt{2}$$

$$J = (\sqrt{2} - 2)(\sqrt{2} + 3) = 2 + 3\sqrt{2} - 2\sqrt{2} - 6 = \sqrt{2} - 4$$

2) Montrons que: $I - J = 5\sqrt{2} + 15$

$$I - J = 11 + 6\sqrt{2} - (\sqrt{2} - 4) = 11 + 6\sqrt{2} - \sqrt{2} + 4 = 5\sqrt{2} + 15$$

3) Montrons que: $I + J = (\sqrt{2} + 3)(2\sqrt{2} + 1)$

$$I + J = (\sqrt{2} + 3)^2 + (\sqrt{2} - 2)(\sqrt{2} + 3) = (\sqrt{2} + 3)(\sqrt{2} + 3 + \sqrt{2} - 2) = (\sqrt{2} + 3)(2\sqrt{2} + 1)$$